An Informal Information Theory Approach to Astronomical Imaging

Illustrated with CCDStack
by
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Imaging intuition / common sense

Light is like sound?
- Sound propagates thru a medium (air) in waves
- Waves exhibit diffraction, refraction, reflection and Doppler effects
- Noise is undesirable sound (e.g. traffic)
- The faintest sounds may be heard with a sensitive enough receiver
- Seeming implications for astro-imaging:
  - High sensitivity (unlimited?) can be achieved by eliminating extraneous noise (sky, camera)
  - Sensitivity becomes nearly linear as the signal significantly exceeds the noise
  - Ever-more sensitive cameras are possible without theoretical limit

What is the medium for light?
- Electromagnetic “Ether” pervades all space?
- Michelson–Morley experiment (1887) challenges Ether concept
- Einstein’s special theory of relativity and the photo-electric effect (1921 Nobel prize)
- Light is profoundly quantum - photons

The light/sound analogy is faulty
- The analogy appears to work for strong signals where light’s quantum nature is hidden (averaged out)
- The basic analogy was accepted for a very long time - abandoned by scientists only about a century ago
- The analogy fails significantly for faint signals that reveal the quantum nature of light, common in astronomical imaging
Light is weird and defies common sense

- Wave-particle duality
  - Double slit experiment
  - Very weird, no one truly “understands” it
    - "Anyone who is not shocked by quantum theory does not understand it." - Niels Bohr
    - "Nobody understands quantum theory." - Richard Feynman
Most of light’s true weirdness is not directly relevant for astro-imaging

- For most optical purposes (telescope design) light is a wave
- For most signal detection purposes, light is a quantum particle – the photon
- A quantum particle (photon) contains a strictly limited amount of information
  - Individual photons are not dividable
  - Individual photons do not carry intensity or contrast information
- The quantum signal is a collection of photons that contains limited information
  - All information from distant objects is conveyed by photons
  - Information is limited by the number of photons
  - Astro-imaging implications:
    - A limited number of photon are emitted and by the object and collected by the aperture
    - An exposure contains limited information and cannot be pushed beyond that information
    - Camera Quantum Efficiency (QE) cannot exceed 100%
    - Ever-more sensitive cameras are not possible beyond 100% QE, we are near the limit
    - Current amateur astro-cams are almost as sensitive as possible (camera with 70% QE with 1.4x more exposure time = 100%)
- Quantum signals contain self-noise
  - Photons are emitted and arrive at random times, like raindrops on the ground
  - Quantum collections (raindrops or light) have intrinsic uncertainty
  - Poisson statistics describes the uncertainty of quantum accumulations
  - Noise = uncertainty
  - Astro-imaging implications:
    - It is not possible to eliminate noise by eliminating other signals (sky, camera)
    - Noise grows with signal, but does not grow as fast as the signal
    - S/N is not linear
Signal = Photons

exp = 50 milliseconds (20 fps)  stack 76,000 exps (1 hr)

H-a filtered ICCD on 14.5” @ f/24
Quantum Imaging - a different paradigm

- **Signal = collection of quantum bits (photons)**
  - Intensity of signal = number of photons
  - Intensity is not an arbitrary number (such as ADU or DN)

- **Noise = Uncertainty**
  - Noise is not a thing, it is an attribute of the signal itself
  - Noise is not other signals (e.g. traffic or sky)
  - Noise (uncertainty) of other signals contributes noise to the signal of interest
  - Noise is a measure of the deviation (“error”) of observations (collections, signals) from the mean (“true” value)
  - Most noises are “normal” – observations form a bell curve
  - One sigma ($\sigma$) is the zone around the mean that encompasses 68% of the observations

- **Signal to Noise Ratio = Signal / Noise**
  - S/N (SNR) characterizes the integrity of the signal
  - Signal intensity alone cannot determine image quality
  - S/N limits ability to discern objects (e.g. limiting mag and contrast detection)
  - S/N has as many definitions and measures as does “signal”
    - Object S/N
    - Sample (pixel) S/N (easy to measure)
    - Informational S/N is fixed at time of exposure (imbedded in image and does not change)
    - Measured S/N (S/STD) is affected by processing transformations
Sigma $\sigma$ = Standard Deviation (STD)
68% chance a measured value is within 1 $\sigma$ of mean, >99% chance within 3 $\sigma$

Poisson/shot noise $\sigma = \sqrt{\text{quantum\_count}}$

If an observed signal = 100 photons (with no other signals or noise) then
$\sigma = 10$: 68% chance the “true” signal is between 90-110; 32% chance true signal is <90 or >110
$3\sigma = 30$: 99% chance the “true” signal is somewhere between 70-130
In the absence of external noise: $S/N = S/\sigma = S/\sqrt{S} = \sqrt{S} = \sigma$
Information Theory approach

- Information theory was developed for electronic transmission and computer analysis (Claude E. Shannon, 1948) then expanded to encompass mathematics and physics.
- Information principles are similar to thermodynamics, especially conservation of information (Hawking was wrong – black holes do not destroy information).
- “Informal astro-imaging information theory” conceptualizes the information content of images based on quantum photon input from astro-objects along with confounding noises.
  - “Informational S/N” describes dynamics of the information fixed in the exposure.
  - Object oriented imaging implements informational S/N to characterize the integrity of information about the object.
  - Image information cannot be increased or decreased after the exposure(s).
- “Informal astro-imaging information theory” also applies to resolution and sampling dynamics.
  - Nyquist theorem.
  - Quadratic convolution of resolution elements (optics, seeing, tracking, etc.).
  - Convolution.
  - Deconvolution.
- Object Information theory is expressed in Object oriented imaging.
- GIGO (Garbage In -> Garbage Out).
Object Oriented Imaging

- Imaged object is the primary metric (not pixels)
- Object can be defined as almost anything:
  - Natural / Real
    - Star
    - Galaxy
    - Nebula
    - Planet or other solar system object
    - Feature within another object (e.g. nebula within galaxy)
  - Constant
    - Fixed angular sky area (e.g. square-arcsec as used for extended object magnitude density)
  - Arbitrary
    - Pixel
    - Image
- Object analysis implements informational S/N to characterize the integrity of object’s image
  - Common application is photometry
    - Object S/N is used to calculate the “margin of error”
    - Photometric limiting mag is $S/N = 3$ (related to $3\sigma = 99\%$ probability that the object is real)
  - Object analysis can determine Contrast Transfer Function or MTF (beyond the scope of this presentation)
- Object analysis is very useful for theoretical investigations of imaging equipment and practices
  - Aperture
  - Focal length, f-ratio and the “$f$-ratio myth”
  - Equipment comparisons (e.g. compare cameras with different pixel pitch)
  - Modes of operation (e.g. binning)
  - Different exposure and sub-exposure times
Pixel Metrics

- Pixel is an object and thus a sub-set of Object oriented imaging
  - Object Oriented Imaging does not conflict with pixel metrics
  - The larger context of Object oriented imaging illuminates issues that are awkward to address with pixel metrics
- Pixel can be used to simplify some analyses (but beware!)
- The pixel object is particular and arbitrary
  - Depends on particular equipment: camera and focal length
  - Changes when image is re-sampled (resized)
  - Cannot be used to qualify or compare equipment or modes without normalization/conversion
- Pixel metrics are common and useful
  - Easy to understand and measure, e.g. pixel S/N = [mean / std] of selected area
    - At least for flat field frames and sky background (same thing)
    - Not so easy for most astro-objects because variations in the object affect STD
  - Marginally useful to characterize superficial appearance of display image
    - Characterize “graininess” of a semi-uniform area in a particular display image (sensitive to scaling)
    - No way to rigorously and meaningfully measure/characterize an entire image (though often spoken as if)
    - Pixel based “image S/N” is essentially a subjective aesthetic judgment
- Simple pixel metrics are inadequate to characterize object images
  - Pixel signal and noise can behave very differently from object metrics
  - For a fixed aperture and time, pixel signal and noise vary by:
    - Camera pixel size
    - Camera binning
    - Focal length
    - Processing (e.g. resizing for display)
  - Pixel S/N alone is meaningless without relating or normalizing to a reference
  - Pixel S/N is often not “informational” and strongly affected by processing (e.g. blur filter)
Measure pixel S/N
Object Signal

- Signal = count of photons from object
- Each photon conveys 1 bit of information about:
  - The existence of the photon itself and time of arrival (during an exposure or not)
  - Location of arrival (translated to plate location in image)
  - Wavelength (color), polarization
- Object signal should be expressed in quantum units
  - Because OOI signal is used in non-linear quadratic equations
  - Arbitrary units (ADU, DN) should be converted to photons
  - For normal CCD and COMOS: photons = electrons = ADU * gain
- Pixels are a grid overlaid on the object to sample information
  - Changing grid size (sample rate) has no effect on the virtual image (object S/N’)
  - Pixel S/N varies with sampling, even if object S/N is constant
  - Sampling is necessary
  - Pixel dynamics are useful and informative when used in proper context
  - Pixel dynamics are misleading when used out of context (e.g. “f-ratio myth”)
- To calculate object signal from pixels:
  Object Signal = sum of photons from all pixels encompassing the object

\[ \sum_{i=1}^{n} S_i \]

\[ S = \text{pixel ADU} \times \text{gain} \]
\[ p = \text{pixel} \]
\[ n = \text{number of pixels encompassing object} \]
Object Noise

- Object Noise = quadratic sum of all noises associated with the object, including instrument effects
  
  \[
  \text{Object noise} = \sqrt{n_1^2 + n_2^2 + n_3^2 + \ldots}
  \]

- Astro-imaging natural noises:
  - Object self-noise = Poisson noise of the collection of object photons
  - Sky noise = Poisson noise of collection of sky photons from the same area and time as the object
  - Poisson noise = \(\sqrt{\text{signal}}\) so:
    
    \[
    \text{Object natural noise} = \sqrt{\text{Object} + \text{Sky}}
    \]
    
    Object = number of photons from object
    Sky = number of photons from sky

- Astro-imaging instrument noises:
  - Object instrument noise = camera and calibration noises from same area and time as the object
  - Noises from pixels encompassing the object:
    
    Instrument pixel noise = \(\sqrt{\text{dc} + \text{rn}^2 + \text{cn}^2}\)
    
    dc = dark current (electrons)
    rn = read noise (electrons RMS)
    cn = calibration noise (dark and flat)
    
    Object instrument noise = \(\sqrt{\text{np} \times \text{pixel\_noise}^2}\)
    
    = \(\sqrt{\text{np}} \times \text{pixel\_noise}\)
    
    np = number of pixels encompassing object in area and time (num exps)
  - Sample rate (pixel size) and number of sub-exp have a strong effect on object instrument noise by varying the number of pixels used to image the object (such affects are not obvious using simple pixel metrics)

- Total object noise = \(\sqrt{\text{object} + \text{sky} + n \times \text{pixel\_noise}}\)
  
  - A significantly larger term renders smaller term(s) insignificant
  - “sky limited” exposures bury pixel\_noise, for example - object = 1 pixel with no signal:
    
    if (sky = 500e-) then sky noise = 22.4e-
    if (sky = 500e- and camera noise = 10e-) then total noise = 24.5e- (almost the same as sky alone)
Object S/N

Object S/N = object signal / object noise

\[
S/N = \frac{\text{Signal}}{\sqrt{\text{Signal} + \text{sky} + \text{npix} \times (\text{dark} + \text{readout}^2)}}
\]
narrowband (sky = 0)
compare sample rates /multi-exposures
Data Rejection

Data rejection exploits signal and noise dynamics to suppress unwanted events and instrument artifacts.

Many data rejection methods necessarily employ object analysis, even if it not labeled or thought of as such (e.g. a hot pixel is only hot relative to an object consisting of a collection of other pixels).

Rejection is a non-destructive, temporary/intermediate stage between detection and interpolation.

CCDStack employs a binary bit map (separate from image data) for rejections.

Permanent rejection or unknowable = “Missing Value”

CCDStack Missing Value = -33,333 ADU
Missing Value is accessible via Pixel Math Compiler.

Single frame data rejections

Reject hot or cold pixels based on object = neighbor pixels in same frame
Reject (mask) consistently bad pixels; possible alternative to dark subtraction
Arbitrary rejections (e.g. grow, freehand draw)

Stack data rejections

Concept: pixel stack
Importance of normalization prior to stack rejection
Identify non-normal pixel stacks to isolate and reject outliers
  • Poisson (works on stacks with as few as 2 frames)
  • STD (does not work well on short stacks)
Reject distribution tails: min/max (median is a special case of min/max)

Interpolation / substitution

Substitute interpolated values from neighbor pixels in the same frame
Substitute interpolated values from other frames in the stack (happens in combine)
Applies to Rejects and Missing Values
Data rejection methods, including median are predicated on the assumption that the various areas in each image have very similar average intensities. If one of more images violates that assumption then data rejection will malfunction and produce a defective final image.
For successful data rejection it is necessary to normalize the images using the slope and intercept concept.

**Normalized (scalar & offset)**
Data Reject: Poisson Sigma Reject

- Poisson noise = square-root(signal) measured in electrons
- Robust on any size stack (as few as 2 images)
- Tends to reject the least number of natural pixels while detecting outliers.
- Susceptible to strange star effects for disparate PSF (limit number of iterations to minimize this effect).
- Best with accurate GAIN and ReadNoise (Camera Manager) but multiplier can compensate.
STD Sigma Reject

- STD = $\sqrt{(P_1-a)^2 + (P_2-a)^2 + ...}$
  - Where:
    - $P_1$ = pixel 1 ADU
    - $P_2$ = pixel 2 ADU ...
    - $a$ = mean of all pixels ($P_1$ thru $P_n$)
- Not robust on small stacks
- Difficult to know appropriate factor (varies with stack size)
- Handles disparate PSF well
Clip Min/Max

- Blend of Mean and Median
- Robust
- Best on large stacks (>12 frames)
- Use conservative parms
- Min & Max do not need to be the same (Max does most of the work)
Data Rejection Techniques

- Verify Rejections over All Images
  - Blink (use info rectangle) – look for over or under rejection
  - Renormalize if too unbalanced (multiple normalization is OK)
  - For stacks registered/re-sampled via methods other than Nearest Neighbor, the registration reference frame will naturally have more rejections
Special Treatments for Outliers

- Special Treatment for Bad Outliers (airplane, etc.)
  - Draw rectangle over the area
  - Apply strong rejection to that area in that image (restrict to selection; apply to this)
  - Grow rejections to capture faint trail edges
  - Use freehand draw to remove stubborn artifacts
Virtual image resolution (FWHM) is the quadratic convolution of resolution elements:
- FWHM = \sqrt{(\text{seeing}^2 + \text{optics}^2 + \text{tracking}^2 + ...)}
- As with S/N quadratic, a larger term can dominate
- Seeing is often the limiting (dominant) term for mid-to-large apertures with FL > 60” (approx)
- Small refractors and short focus reflectors usually have large angular optical spots that limit true resolution and are thus somewhat immune to seeing
- AO and high speed imaging remove some or most coherent image wander due to seeing (and mount)

Nyquist sampling criterion
- Nyquist = function width (std) of intensity function (not “2”, as is often asserted)
- Nyquist = FWHM 2.355 pixels for 1-dimensional Gaussian PSF
- Nyquist = FWHM 3.33 pixels for diagonal of 2-dimensional Gaussian PSF
- FWHM > 3.5 includes headroom for non-Gaussian PSF, resampling and deconvolution

Under-sampling
- Wide FOV images are often undersampled and visa versa
- Under-sampling can improve S/N due to using fewer (noisy) pixels per object (can be important for narrow band depth)
- Under-sampled images fail to capture the full resolution of the virtual image
- Under-sampled images are pseudo-sharp; “pin point” stars are a symptom of undersampling
- Under-sampling acerbates star saturation (fewer pixels fill faster)
- Under-sampled stars can mimic hot pixels and visa versa, so offset acquisition (dithering) is important

Over-sampling
- Over-sampled images tend to have smaller FOV, but many objects are not large
- Over-sampling can diminish S/N due to using more (noisy) pixels per object; “sky limited” exposures take longer but are important
- Oversampled images may look “soft” but contain superior resolution and can often be meaningfully deconvolved to yield even more resolution
Virtual image
Sampling grids
Sampled images
Resampling = Information Transform

- Resampling changes pixilation with usually minimal affect on object metrics
- Pixel S/N changes but object S/N is essentially unaffected other than the information entropy introduced by under-sampling
- Down sampling convolves information; excessive undersampling damages objects
- Registration is 2 part:
  - Alignment
    - Object oriented: stars and star associations (Star Snap, Star Match, CCDIs)
    - Pixel based: FFT, Manual, (OLS in development)
  - Resample
- Primary resampling transforms (affine transformation matrix)
  - Shift
  - Rotate
  - Scale (magnify/shrink)
- Resample methods
  - Nearest Neighbor (preserves noise profile, preserves Missing Values)
  - Bi-linear (good shift but not optimal for rotation or scale)
  - Bi-cubic B-spline (good rotation, very smooth, small degradation)
  - Quadratic B-spline (sharp with minimal ringing)
  - Mitchell (good enlarge/shrink)
  - Lanczos/sinc 256 (each resampled pix is calculated from 256 original pix)
“Conservation of information” allows conversion of resolution to S/N
Convolution = smoothing, blurring, ”noise reduction”
Convolution increases apparent and measured S/N
The apparent S/N increase does not increase actual information; not possible to deepen and exposure via convolution
Low pass / blur kernel filters:
  • Mean
  • Median
  • Mode
  • Gaussian
CCDStack uses circular aperture for kernel convolutions
“Conservation of information” permits a sort of conversion of S/N to resolution

- Raw image contains convolved information (e.g. seeing blur)
- There is no single mathematical function to undo convolved information
  - Some convolutions are effectively unrecoverable (e.g. all pixels = 0 is seemingly a non-conserving transformation)
  - Scope and seeing convolutions can be characterized via point spread function (PSF) derived from star image
- Un-sharp masking is a fast and relatively simple form of de-convolution
- Most deconvolution algorithms (L/R, MaxEnt) must repeatedly iterate to converge on a good solution using PSF
- S/N (noise) affects convergence and limits application
  - Use unsharp mask controls to avoid excess noise
  - CCDStack implements noise-limited deconvolution models
  - Employ layering to blend low S/N (dim) with sharpened high S/N (bright)
Several thousand iterations of CCDStack MaxEnt deconvolution followed by unsharp mask with gamma-DDP scaling.
Light is a quantum phenomenon that defies common sense in the dim domain of DS astro-imaging
Photons provide our only information from astro-objects
Images are strictly limited by information from photon accumulations (GIGO)
Noise = uncertainty, measures of noise characterize the magnitude of certainty
The light signal itself contains intrinsic noise (uncertainty)
Object oriented imaging characterizes an image by how well it represents object(s) regardless of pixel scale, f-ratio, etc.
Pixel S/N is a common and seemingly simple concept but it is inadequate and easily misleading
Object S/N provides a deeper understanding of astro-imaging dynamics and is a powerful tool for analysis, comparisons and predictions
Information can be transformed (within limits)
CCDStack was designed and continues to develop from an information / object oriented approach
Thank You!